Zusammenfassung HF II

Wellenmatrizen

Streumatrix: Def.: $\underline{U}_r = \underline{\underline{S}} \cdot \underline{U}_h$

Eigenschaften:
- Reziprozität: $\underline{\underline{S}} = \underline{\underline{S}}^T \Leftrightarrow S_{ij} = S_{ji}$
- Verlustlos: $\|S\|^{-1} = \|S\|^{*}$ $\Leftrightarrow \sum_{\mu=1}^{n} |S_{\mu\nu}|^2 = 1$

Für $Z_{L1} \neq Z_{L2}$:

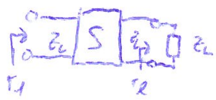
$$a_i = \frac{U_{hi}}{\sqrt{Z_{Li}}} \qquad b_i = \frac{U_{ri}}{\sqrt{Z_{Li}}}$$

$\rightarrow \underline{b} = \underline{\underline{S}}\,\underline{a}$

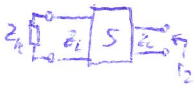
(r_k auf jeweiligen Wellenwiderstand anpassen)

Betriebsverhalten: $\underline{r}_a := \dfrac{Z_a/Z_L - 1}{Z_a/Z_L + 1}$

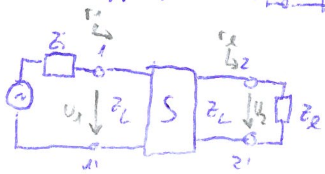
bei zwischengeschaltetem VP: $r_1 = S_{11} + \dfrac{S_{12}S_{21}r_\ell}{1 - S_{22}r_\ell}$



bzw. $r_2 = S_{22} + \dfrac{S_{12}S_{21}r_h}{1 - S_{11}r_h}$



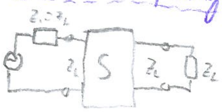
Spg-Übertragung: $\dfrac{U_2}{U_1} = \dfrac{S_{21}(1 + r_\ell)}{(1 - S_{22}r_\ell)(1 + r_1)}$



[Beachte: $r_1 = S_{11} + \dfrac{S_{12}S_{21}r_\ell}{1 - S_{22}r_\ell}$]

Betriebsleistungsverstärkung: $g_T =: \dfrac{P_2}{P_{i\,max}} = \dfrac{|S_{21}|^2 (1 - |r_i|^2)(1 - |r_\ell|^2)}{|(1 - S_{11}r_i)(1 - S_{22}r_\ell) - S_{12}S_{21}r_\ell r_i|^2} = |A_B|^2$

Wellenwiderstandsanpassung:



$U_{h1} = \dfrac{U_0}{2}$ $\qquad r_1 = S_{11}$

$\dfrac{U_2}{U_1} = \dfrac{S_{21}}{1 + S_{11}}$ $\qquad r_2 = S_{22}$ $\qquad g_T = |S_{21}|^2$

Transmissionsmatrix: Def.: $\begin{pmatrix} U_{r1} \\ U_{h1} \end{pmatrix} = \underline{\underline{T}} \cdot \begin{pmatrix} U_{h2} \\ U_{r2} \end{pmatrix}$

$\rightarrow$ Bei Kettenschaltungen: $\begin{pmatrix} U_{r1} \\ U_{h1} \end{pmatrix} = \prod_{i=1}^{n} \underline{\underline{T}}_i \cdot \begin{pmatrix} U_{h2} \\ U_{r2} \end{pmatrix}$

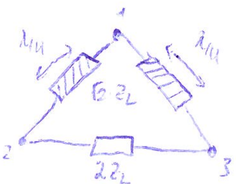
(Vorsicht: num. Instabilitäten möglich)

Umrechnung: $\underline{\underline{T}} = \dfrac{1}{S_{21}} \begin{bmatrix} 1 & +S_{11} \\ S_{22} & -\det \underline{\underline{S}} \end{bmatrix}$

$\underline{\underline{S}} = \dfrac{1}{T_{22}} \begin{bmatrix} T_{12} & \det \underline{\underline{T}} \\ 1 & -T_{21} \end{bmatrix}$

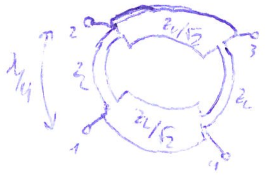
Leitungsschaltungen

Wilkinson-Teiler:



$$\underline{\underline{S}} = -\frac{j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

90°-Hybridkoppler:

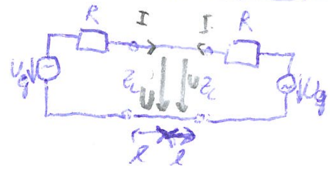


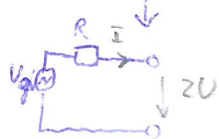
$$\underline{\underline{S}} = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & j \\ 0 & 0 & j & 1 \\ 1 & j & 0 & 0 \\ j & 1 & 0 & 0 \end{bmatrix}$$

Hybrid: 4-Tor, alle Tore mit Z_L abgeschlossen

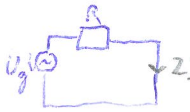
Even-/Odd-Mode-Analysis

even:





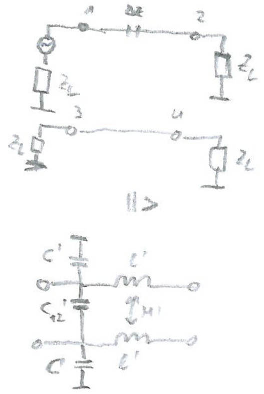
odd:



Gekoppelte Ltg.: Kopplungsfaktor $k = \dfrac{M'}{L'} = \dfrac{C_{12}'}{C' + C_{12}'} = \dfrac{Z_{Le} - Z_{Lo}}{Z_{Le} + Z_{Lo}}$

$$Z_L = \sqrt{Z_{Le} \cdot Z_{Lo}} \qquad Z_{Le} = Z_L \sqrt{\frac{1+k}{1-k}} \qquad Z_{Lo} = Z_L \sqrt{\frac{1-k}{1+k}}$$

Richtkoppler: Bed. für Auslöschung an Tor 4: $\frac{M'}{Z_L^2} = C_{12}'$?



$$S_{21} = \frac{jk\sin(\beta l)}{\sqrt{1-k^2}\cos(\beta l) + j\sin(\beta l)}$$

$$S_{31} = \frac{\sqrt{1-k^2}}{\sqrt{1-k^2}\cos\beta l + j\sin(\beta l)}$$

$$\underline{\underline{S}} = \begin{bmatrix} 0 & k & -j\sqrt{1-k^2} & 0 \\ k & 0 & 0 & -j\sqrt{1-k^2} \\ -j\sqrt{1-k^2} & 0 & 0 & k \\ 0 & -j\sqrt{1-k^2} & k & 0 \end{bmatrix}$$

Koppeldämpfung: $a_k = -10\log\frac{P_2}{P_{1max}} = -20\log|S_{21}|$

Durchgangsdämpfung: $a_B = -20\log\frac{P_3}{P_{1max}} = -20\log|S_{31}|$

Richtdämpfung: $a_R = -10\log\frac{P_4}{P_2} = -20\log\left|\frac{|S_{41}|}{|S_{21}|}\right|$

[3dB-Richtkoppler: $k = \frac{1}{\sqrt{2}}$]

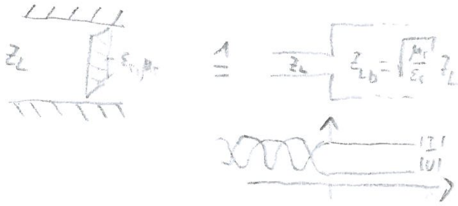
$$\rightarrow S = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & -j & 0 \\ 1 & 0 & 0 & -j \\ -j & 0 & 0 & 1 \\ 0 & -j & 1 & 0 \end{bmatrix}$$

Reflexion ebener Wellen an Grenzflächen

Senkrechter Einfall

Dielektrikum:

$$\Gamma(0) = \frac{\sqrt{\frac{\mu_r}{\varepsilon_r}} - 1}{\sqrt{\frac{\mu_r}{\varepsilon_r}} + 1}$$

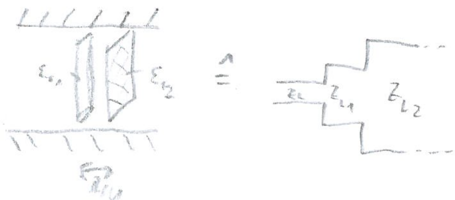


$\lambda/4$-Absorber: $(\mu_r = 1)$

$$Z = \frac{Z_{L1}^2}{Z_{L2}} \doteq Z_L$$

$\rightarrow Z_{L1} = \sqrt{Z_L \cdot Z_{L2}}$

$\rightarrow \varepsilon_{r1} = \sqrt{\varepsilon_{r2}}$



Leitende Ebene:

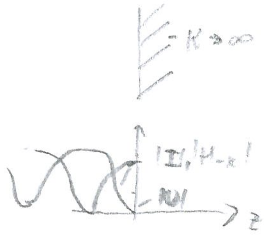
$\vec{J}_F(0) = \vec{n} \times \vec{H}(0)$

$\Gamma(0) = -1 \rightarrow U_r = -U_h$

$\rightarrow E_r = -E_h$ $[E_z = 0]$

$H_{-x}(0) = 2 \cdot H_h = 2\frac{E}{Z_{F0}}$

[im Leiter: Skineffekt]

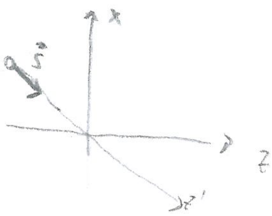


Schräger Einfall

Ebene Welle

$\vec{E}_y = \vec{E}_h e^{-j\beta_0 z'}$

$\vec{H} = \frac{E_y}{Z_{F0}}$ mit $z' = -x\cos\alpha + z\sin\alpha$



$\rightarrow \lambda_x = \frac{\lambda_0}{\cos\alpha}$ $\lambda_z = \frac{\lambda_0}{\sin\alpha}$

$\rightarrow v_{Px} = \frac{c_0}{\cos\alpha}$ $v_{Pz} = \frac{c_0}{\sin\alpha}$ ... Phasengeschwindigkeit

$v_{Ex} = c_0\cos\alpha$ $v_{Ez} = c_0\sin\alpha$ ... Energiegeschwindigkeit

$\left.\right\}$ $v_{Pz} v_{Ez} = c_0^2$ (gilt nicht i. A.)

1) E parallel zu leitender Ebene (TM)

$\alpha_1 = \alpha_2$

$\underline{E}_r = -\underline{E}_h$ (bei $y = 0$)

$\underline{E}_y = \underline{E}_0 \sin\frac{2\pi x}{\lambda_x} e^{-j\frac{2\pi z}{\lambda_z}}$ mit $\underline{E}_0 = 2j\underline{E}_H$

$\underline{H}_{-x} = \frac{E_y}{Z_{F0}/\sin\alpha}$

$\underline{H}_z = j\frac{E_0}{Z_{F0}/\cos\alpha}\cos\frac{2\pi x}{\lambda_x} e^{-j\frac{2\pi z}{\lambda_z}}$

$\vec{S} = \frac{1}{2}\frac{|E_y|^2}{Z_{F0}/\sin\alpha}\vec{z}$

$Z_{FH} = \frac{Z_{F0}}{\sin\alpha}$

2) H parallel zu leitender Ebene

$E_r = E_h$ (bei $y = 0$)

$E_y = 2E_h\sin\alpha\cos\left(\frac{2\pi y}{\lambda_y}\right)e^{-j\frac{2\pi z}{\lambda_z}}$

$E_z = 2j E_h\cos\alpha\sin\left(\frac{2\pi y}{\lambda_y}\right)e^{-j\frac{2\pi z}{\lambda_z}}$

$H_x = 2\frac{E_h}{Z_{F0}}\cos\left(\frac{2\pi y}{\lambda_y}\right)e^{-j\frac{2\pi z}{\lambda_z}}$

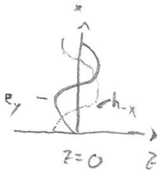
$\vec{S} = \frac{1}{2}\frac{|E_y|^2}{Z_{F0}\sin\alpha}\vec{z}$

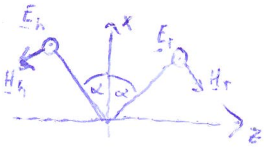
$Z_{FE} = Z_{F0}\sin\alpha$

Schräger Einfall

auf Leiter:

*) E parallel (TE-Welle)





$\underline{E}_r = -\underline{E}_h$ (bei $x=0$)

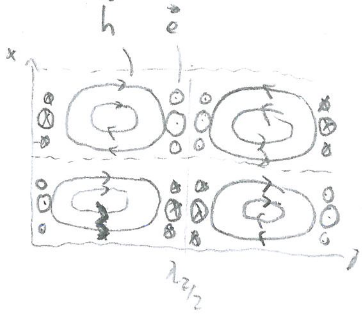
$\lambda_x = \frac{\lambda_0}{\cos\alpha}$

$\underline{E}_y = 2j\,E_H \sin\frac{2\pi x}{\lambda_x}\, e^{-j\frac{2\pi z}{\lambda_z}}$

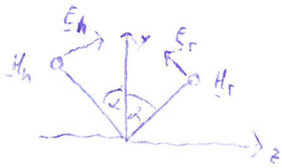
$\underline{H}_{-x} = \frac{E_y}{Z_{F0}/\sin\alpha}$

$\underline{H}_z = j\,\frac{E_0}{Z_{F0}/\cos\alpha}\cos\frac{2\pi x}{\lambda_x}\, e^{-j\frac{2\pi z}{\lambda_z}}$

$\vec{S} = \frac{1}{2}\frac{|E_y|^2}{Z_{F0}/\sin\alpha}\cdot\vec{z}$ $\quad(\to Z_{FH} = \frac{Z_{F0}}{\sin\alpha})$



*) H parallel (TM-Welle)



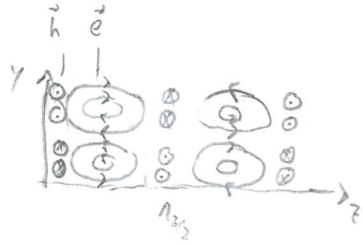
$\underline{E}_r = \underline{E}_h$

$\lambda_z = \frac{\lambda_0}{\sin\alpha}$ $\quad \lambda_y = \frac{\lambda_0}{\cos\alpha}$

$\underline{E}_y = 2\,E_h \sin\alpha\cos\frac{2\pi y}{\lambda_y}\, e^{-j\frac{2\pi z}{\lambda_z}}$

$\underline{E}_z = 2j\,E_h \cos\alpha\sin\frac{2\pi y}{\lambda_y}\, e^{-j\frac{2\pi z}{\lambda_z}}$

$\underline{H}_{-x} = 2\,\frac{E_h}{Z_{F0}}\cos\frac{2\pi y}{\lambda_y}\, e^{-j\frac{2\pi z}{\lambda_z}}$



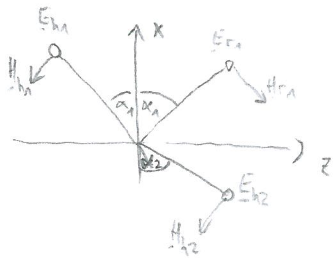
Dielektrische Grenzschicht:

$\frac{E_{h1}}{H_{h1}} = \frac{E_{r1}}{H_{r1}} = Z_{F0}\cdot\frac{1}{\sqrt{\varepsilon_{r1}}}$

$\frac{E_{h2}}{H_{h2}} = \ldots = Z_{F0}\,\frac{1}{\sqrt{\varepsilon_{r2}}}$

$\lambda_1 = \frac{\lambda_0}{\sqrt{\varepsilon_{r1}}}$ $\quad \lambda_2 = \frac{\lambda_0}{\sqrt{\varepsilon_{r2}}}$

$\lambda_z = \frac{\lambda_0}{\sqrt{\varepsilon_{r1}}\cdot\sin\alpha_1} = \frac{\lambda_0}{\sqrt{\varepsilon_{r2}}\sin\alpha_2}$



*) E ∥ :

$\underline{r} = \frac{E_{r1}}{E_{h1}} = \frac{\sin(\alpha_2-\alpha_1)}{\sin(\alpha_2+\alpha_1)}$ $\qquad \frac{\sin\alpha_2}{\sin\alpha_1} = \sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}$

$\underline{t} = \frac{E_{h2}}{E_{h1}} = \frac{2}{1+\frac{\cos\alpha_2}{\cos\alpha_1}\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}}$

*) H ∥ :

$\underline{r} = \frac{\sin 2\alpha_1 - \sin 2\alpha_2}{\sin 2\alpha_1 + \sin 2\alpha_2}$ $\qquad \frac{\sin\alpha_2}{\sin\alpha_1} = \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}$

$\underline{t} = \frac{2\sin\alpha_2\cos\alpha_1}{\sin(\alpha_1+\alpha_2)\cos(\alpha_1-\alpha_2)}$

Hohlleiter mit Rechteckquerschnitt

H-Wellen

Im Abstand $m \cdot \frac{\lambda_x}{2}$ leitende Wand einziehen → H_{m0}-Welle

H_{mn}-Welle:

$$\underline{E}_z = 0$$

$$\underline{H}_z = H_0 \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{-j\beta z}$$

$$\underline{H}_y = j\frac{\beta}{\beta_c^2}\frac{n\pi}{b} \underline{H}_0 \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{-j\beta z}$$

$$\underline{H}_x = j\frac{\beta}{\beta_c^2}\frac{m\pi}{a} \underline{H}_0 \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{-j\beta z}$$

$$\underline{E}_y = -\underline{H}_x \cdot Z_{FH}$$

$$\underline{E}_x = \underline{H}_y \cdot Z_{FH}$$

$$\beta_c = \frac{2\pi}{\lambda_c} \qquad \lambda_c = \frac{1}{\sqrt{\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2}}$$

$$Z_{FH} = \overset{120\pi\,\Omega}{\overset{\Vert}{Z_{F0}}} \cdot \frac{\lambda_H}{\lambda_0}$$

$$\boxed{\lambda_H = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}}$$

∀ Hohlleiterwellen gültig

$$\beta_{cx} = \frac{m\cdot\pi}{a} \qquad \beta_{cy} = \frac{n\pi}{b} \qquad \beta_{cx}^2 + \beta_{cy}^2 = \beta_c^2$$

H_{10}-Welle: ($\lambda_c = 2a$)

$$\underline{E}_y = E_0 \sin\frac{\pi x}{a} e^{-j\beta z}$$

$$\underline{H}_x = \frac{\underline{E}_y}{Z_{FH}}$$

$$H_z = j\frac{E_0}{Z_{F0}}\frac{\lambda_0}{\lambda_c}\cos\frac{\pi x}{a} e^{-j\beta z}$$

$$v_p = c_0 \cdot \frac{\lambda_H}{\lambda_0} = f\cdot\lambda_H$$

$$v_E = \frac{c_0^2}{v_p}$$

E-Wellen

E_{mn}-Welle:

$$\underline{H}_z = 0$$

$$\underline{E}_z = \underline{E}_0 \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{-j\beta z}$$

$$\underline{E}_x = -j\frac{\beta}{\beta_c^2}\frac{m\pi}{a} \underline{E}_0 \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{-j\beta z}$$

$$\underline{E}_y = -j\frac{\beta}{\beta_c^2}\frac{n\pi}{b} \underline{E}_0 \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{-j\beta z}$$

$$\underline{H}_x = -\frac{\underline{E}_y}{Z_{FE}}$$

$$\underline{H}_y = \frac{\underline{E}_x}{Z_{FE}}$$

$$\beta_c = \frac{2\pi}{\lambda_c} \qquad \lambda_c = \frac{1}{\sqrt{\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2}}$$

$$Z_{FE} = Z_{F0} \cdot \frac{\lambda_0}{\lambda_H}$$

Eigenschaften d. H_{10}-Welle:

H_{10} … magnet. Grundwellentyp

Eindeutigkeit:

$$f_{c10} = \frac{c_0}{\lambda_{c10}} = \frac{c_0}{2a}$$

$H_{10}: \frac{\lambda_0}{a} \le 2 \quad H_{01}: \frac{\lambda_0}{a} \le \frac{2b}{a}$

$H_{20}: \frac{\lambda_0}{a} \le 1 \quad H_{02}: \frac{\lambda_0}{a} \le \frac{b}{a}$

H_{10} eindeutig im Bereich $1 \le \frac{\lambda_0}{a} \le 2$

(praktisch: $1{,}053 \le \frac{\lambda_0}{a} \le 1{,}6$)

Stromweise:

$$\vec{j}_F = \vec{n} \times \vec{h}(0)$$

Analogie Ltg.-Theorie:

$$Z_L(H) = \sqrt{\frac{L_s'}{C'}} \cdot \frac{1}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$\beta = \underbrace{\omega\sqrt{L_s' C'}}_{\beta_0} \sqrt{1-\frac{f_c}{f}}$$

Wandverluste:

$$\alpha = \frac{R_\square}{Z_{F0}} \frac{\frac{1}{b} + \frac{2}{a}\left(\frac{\lambda_0}{2a}\right)^2}{\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}}$$

$$\Vert$$

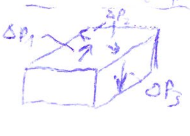
$$-\frac{1}{2}\frac{dP}{dz}\cdot\frac{1}{P}$$

Leistungstransport:

$$S_z = \frac{1}{2}\frac{|E_0|^2}{Z_{F0}}\left(\sin\frac{\pi x}{a}\right)^2$$

$$P = \frac{ab}{4}\frac{|E|^2}{Z_{FH}}$$

Verluste im Hohlleiter



$$\Delta P_1 = -\frac{1}{4} R_\square a \cdot \left|\frac{E_0}{Z_{FH}}\right|^2 \cdot \Delta z \sim \sqrt{f}$$

$$\Delta P_2 = -\frac{1}{4} R_\square a \cdot \left|\frac{E_0}{Z_{F0}}\right|^2 \frac{\lambda_0^2}{(2a)^2} \Delta z$$

$$\Delta P_3 = -\frac{1}{2} R_\square b \frac{|E_0|}{Z_{F0}^2}\frac{\lambda_0^2}{2a}$$

$\Delta P_2, \Delta P_3 \sim f^{-\frac{3}{2}}$

mit $R_\square = \sqrt{\frac{\omega\mu_0}{2\kappa}}$

Hohlleiter mit Kreisquerschnitt

Maxwell'sche Gleichungen in Zylinderkoordinaten

$$\frac{\partial H_z}{\rho\,\partial\varphi} - \frac{\partial H_\varphi}{\partial z} = j\omega\varepsilon_0 E_\rho \qquad \frac{\partial E_z}{\rho\,\partial\varphi} - \frac{\partial E_\varphi}{\partial z} = -j\omega\mu_0 H_\rho$$

$$\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho} = j\omega\varepsilon_0 E_\varphi \qquad \frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho} = -j\omega\mu_0 H_\varphi$$

$$\frac{1}{\rho}\left[\frac{\partial(\rho H_\varphi)}{\partial\rho} - \frac{H_\rho}{\partial\varphi}\right] = j\omega\varepsilon_0 E_z \qquad \frac{1}{\rho}\left[\frac{\partial(\rho E_\varphi)}{\partial\rho} - \frac{\partial E_\rho}{\partial\varphi}\right] = -j\omega\varepsilon_0 H_z$$

Wellen in Hohlleiter mit Kreisquerschnitt

H-Wellen:

$E_z = 0$

$$Z_{FH} = \frac{-E_{\varphi 0}}{H_{\rho 0}} = \frac{E_{\rho 0}}{H_{\varphi 0}} = \frac{\omega\mu_0}{\beta} = Z_{F0}\cdot\frac{\lambda_H}{\lambda_0}$$

$$H_z = H_0 J_m(\beta_c \rho)\cos(m\varphi - \varphi_0)\, e^{-j\beta z}$$

H_{11}:

$\lambda_c = 1{,}71\,D$

$$H_z = H_0 J_1(\beta_c\rho)\cos(\varphi-\varphi_0)\, e^{-j\beta z}$$

$$H_\varphi = j\frac{\beta}{\beta_c^2\rho} H_0 J_1(\beta_c\rho)\sin(\varphi-\varphi_0)\, e^{-j\beta z}$$

$$H_\rho = -j\frac{\beta}{\beta_c} H_0 J_1'(\beta_c\rho)\cos(\varphi-\varphi_0)\, e^{-j\beta z}$$

$$E_\varphi = -H_\rho Z_{FH}$$

$$E_\rho = H_\varphi Z_{FH}$$

Bilder

H_{01}:

$\lambda_c = 0{,}82\,D$

$$H_z = H_0 J_0(\beta_c\rho)\, e^{-j\beta z}$$

$$H_\rho = -j\frac{\beta}{\beta_c} H_0 J_0'(\beta_c\rho)\, e^{-j\beta z}$$

$$E_\varphi = -H_\rho Z_{FH}$$

E-Wellen: allg.: $H_z = 0$

(E_{mn})

$$E_z = E_0 J_m(\beta_c\rho)\cos(m\varphi-\varphi_0)\, e^{-j\beta z}$$

$$E_\varphi = j\frac{m\beta}{\beta_c^2\rho} E_0 J_m(\beta_c\rho)\sin(m\varphi-\varphi_0)\, e^{-j\beta z}$$

$$E_\rho = -j\frac{\beta}{\beta_c} E_0 J_m'(\beta_c\rho)\cos(m\varphi-\varphi_0)\, e^{-j\beta z}$$

$$H_\varphi = \frac{E_\rho}{Z_{FE}}$$

$$H_\rho = -\frac{E_\varphi}{Z_{FE}}$$

(β_c aus d. n-ten Nullstelle von $J_m(\beta_c \frac{D}{2}) = 0$)

E_{01}:

$\lambda_c = 1{,}31\,D$

$$E_z = E_0 J_0(\beta_c\rho)\, e^{-j\beta z}$$

$$E_\rho = -j\frac{\beta}{\beta_c} E_0 J_0'(\beta_c\rho)\, e^{-j\beta z}$$

$$H_\varphi = \frac{E_\rho}{Z_{FE}}$$

Hin- & rücklaufende Wellen im Rechteckhohlleiter

$$E_{yh} = E_h \sin\frac{\pi x}{a}\, e^{-j\beta z}$$

$$H_{xh} = -\frac{E_{yh}}{Z_{FH}}$$

$$H_{zh} = jE_h \underbrace{\frac{\lambda_0}{Z_{F0}\lambda_c}}_{K} \cos\frac{\pi x}{a}\, e^{-j\beta z}$$

$$E_{yr} = E_r \sin\frac{\pi x}{a}\, e^{j\beta z}$$

$$H_{xr} = \frac{E_{yr}}{Z_{F0}}$$

$$H_{zr} = -jE_r K\cdot\cos\frac{\pi x}{a}\, e^{j\beta z}$$

Hohlleiter unterhalb d. krit. Frequenz

$$\lambda_H = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} \to \mathbb{C} \to \beta = (+)\, j\, \frac{2\pi}{\lambda_0}\sqrt{\left(\frac{\lambda_0}{\lambda_c}\right)^2-1} = j\alpha$$

$$\to \alpha = \frac{2\pi}{\lambda_0}\sqrt{\left(\frac{\lambda_0}{\lambda_c}\right)^2-1} = \frac{2\pi}{\lambda_c}\sqrt{1-\left(\frac{\lambda_c}{\lambda_0}\right)^2}$$

$$E(z) = E_0 \cdot e^{-\alpha\cdot\Delta z}$$

$$\lim_{f_0\to 0}\alpha = \frac{2\pi}{\lambda_c}$$

H-Felder: $Z_{FH} = Z_{F0}\cdot\frac{\lambda_H}{\lambda_0} = jZ_{F0}\frac{1}{\sqrt{\left(\frac{\lambda_0}{\lambda_c}\right)^2-1}}$ → 90° Phasenversch. zw. Querfeldstärken → kein Leistungstransport möglich

Anwendungen

[Unbelastete Güte einiger Hohlraumresonatoren:

Quader (H_{101}-Resonanz): $Q_U = \frac{\lambda_R}{\delta}\frac{b(a^2+c^2)^{3/2}}{2[c^2(a+2b)+a^3(c+2b)]}$

$\delta = \sqrt{\frac{2}{\omega\kappa\mu}}$

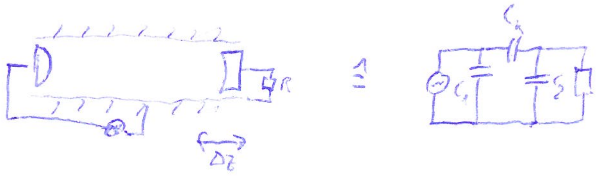
Zylinder (E_{101}-Resonanz): $Q_U = 0{,}38\frac{\lambda_R}{\delta}\frac{1}{1+\frac{1}{2}\frac{D}{c}}$

Zylinder (H_{011}-Resonanz): $Q_U = 0{,}61\frac{\lambda_R}{\delta}\frac{\left[1+0{,}17\left(\frac{D}{c}\right)^2\right]^{3/2}}{1+0{,}17\left(\frac{D}{c}\right)^3}$

HF-Abschwächer:

Leitungsschwächer: $f \ll f_{cH10}$ bzw. $\lambda_0 \gg \lambda_{c,H10} = 2a$

Kapaz. Spg-Teiler:



$$\alpha\,\Delta z \overset{f\ll f_c}{=} \alpha_c\cdot\Delta z = \frac{2\pi}{\lambda_c}\Delta z = 4{,}8\frac{\Delta z}{D} \quad [Np]$$

Mikrowellen-Resonatoren

Hohlraum mit leitender Hülle: H_{mnp}, E_{mnp}

Anzahl d. Halbwellen

$$\lambda_R = \frac{\lambda_c}{\sqrt{1+\left(p\frac{\lambda_c}{2c}\right)^2}}$$ (für Rechteck- und Rundhohlleiter)

Rechteckquader: $\lambda_{c\,m,n} = \frac{1}{\sqrt{\left(\frac{m}{2a}\right)^2+\left(\frac{n}{2b}\right)^2}} \to \lambda_R = \frac{1}{\sqrt{\left(\frac{m}{2a}\right)^2+\left(\frac{n}{2b}\right)^2+\left(\frac{p}{2c}\right)^2}}$

(niedr. Resonanz: H_{101})

Bei E-Wellen: $p=0$ möglich → $\lambda_R = \lambda_c$, $\lambda_H \to \infty$

Unbelastete Güte: $Q_U = \frac{\omega_R\cdot W_{max}}{P_V}$ → max. in E/H gespeicherte Energie; → Wandverluste

→ über eine Halbwelle, 28.05.25

$$W_{max,e} = \int_V \frac{1}{2}\varepsilon_0|E_{max}|^2\, dV$$

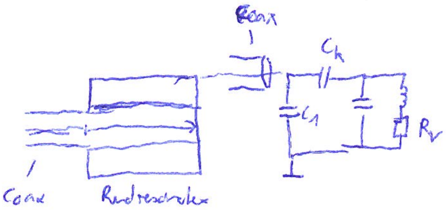
$$W_{max,m} = \int_V \frac{1}{2}\mu_0|H_{max}|^2\, dV$$

[z.B. H_{101}: $W_{el,max} = \frac{1}{2}\varepsilon_0|E_h|^2 abc$]

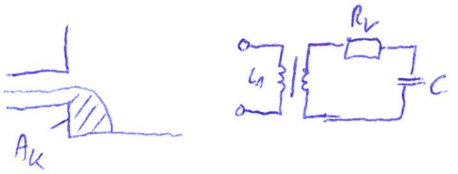
Zusammenfassung HF II

Ankopplung

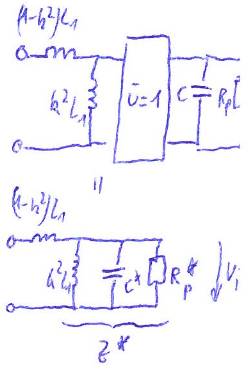
kapazitiv:



induktiv:

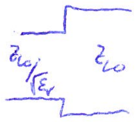


$\hat{=}$



$$A_k = \frac{\lambda_{HR}}{4\pi}\sqrt{\frac{R_p^* \cdot w_R\, \varepsilon_0\, a b c}{Q_U}}$$

Dielektrische Resonatoren



$r = \frac{\sqrt{\varepsilon_r}-1}{\sqrt{\varepsilon_r}+1} \xrightarrow{\varepsilon_r \to \infty} 1 \;\hat{=}$ Leerlauf

$\hat{=}$ magnet. Wand $\mu_r \to \infty$

$\to H_t = 0,\ E_n = 0$

$\to$ Duale Felder: $E_1 \multimap H_2$ (Vertausche μ_r/κ)

$H_1 \multimap -E_2 \quad \Rightarrow E_{010} \hat{=} H_{010}$)

H_{01S}-Resonanz: kleiner Teil (S) d. Feldes im freien Raum

$$\lambda_R(H_{01S}) = \lambda_R(H_{020}) \cdot \Psi\left(\varepsilon_r, \frac{D}{c}\right)$$

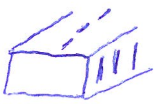
mit $\Psi\left(\varepsilon_r, \frac{D}{c}\right) \approx \frac{1}{1{,}02 + 0{,}15\frac{D}{c}}$

$$Q_U \lessapprox \frac{1}{\tan \delta_\varepsilon}$$

Hohlleiter-Schaltungen

Übertragungsstrecke

Modenfilter:



E-Mode: Längsströme $b \to$ Dämpfung

H_{20}-Moden: Querströme $a \to$ %

H_{01}-Mode: Längsströme $b \to$ %

Anwendung Leitungstheorie

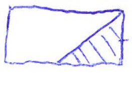
$E_y \multimap U$ (bzw. $E_h \multimap U_h$
$H_{-x} \multimap I$ $\quad E_r \multimap U_r$)

$$\boxed{I_h} = \int_{x=0}^{a} y_{x-z}\, dx = \boxed{\frac{E_h}{Z_{FH}}\frac{2a}{\pi}}$$

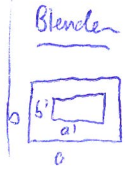
$Z_L = \frac{\pi^2}{8} \cdot \frac{b}{a} \cdot Z_{FH}$ ... Ltg-WW d. Hohlleiters

Bauelemente

Absorber:



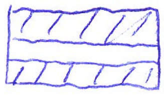
Blenden



$$\lambda_{RB} = 2a\sqrt{\frac{\left(\frac{a'b}{ab'}\right)^2 - 1}{\left(\frac{b}{b'}\right)^2 - 1}}$$

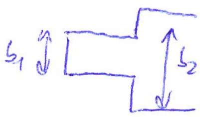
Blenden: $\lambda_{RB} = 2a\sqrt{\frac{\left(\frac{a'b}{ab'}\right)^2-1}{\left(\frac{b}{b'}\right)^2-1}}$ $\qquad Y_{rel} = j\cdot B\cdot \frac{\frac{\lambda_{RB}}{\lambda_0}-\frac{\lambda_0}{\lambda_{RB}}}{\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}}$

→ Kapazitive Blende: $\qquad Y_{rel} = jB\frac{2a}{\lambda_H}$



→ Induktive Blende: $\qquad Y_{rel} = -j\frac{\lambda_H}{a}\cot^2\left(\frac{\pi}{2}\frac{a'}{a}\right)$

Querschnittssprung:



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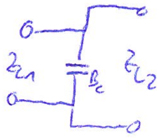
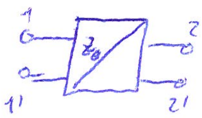


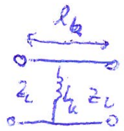
$$Z_i = \frac{\pi^2}{8} Z_{FH}\frac{b_i}{a}$$

Magic Tee:

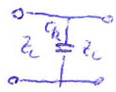
$$\underline{S} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Impedanzinverter:





$X_k = \omega L_k$



$X_k = -\frac{1}{\omega C_k}$

$$\frac{X_k}{Z_L} = \frac{Z_0/Z_L}{1-(Z_0/Z_L)^2}$$

$$\frac{l_k}{\lambda} = -\frac{1}{2\pi}\operatorname{arctan}\frac{2X_k}{Z_L}$$

$$Z_0 = Z_L\cdot\tan\left|\frac{\pi l_k}{\lambda}\right|$$